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Shovan Chowdhury [1]

[1] Associate Professor, Quantitative Methods and Operations Management, Indian Institute of Management, Kozhikode, IIMK Campus PO, Kunnamangalam, Kozhikode, Kerala 673570 India, Email: shovanc@iimk.ac.in, Phone Number (+91) 495 – 2809119

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Shovan Chowdhury*
Indian Institute of Management, Kozhikode
Quantitative Methods and Operations Management Area
Kerala, India
Email: meetshovan@gmail.com

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Abstract

Exponential and Lindley distributions are quite effective in analyzing positively skewed data. While the distributions exhibit some of the distinguishable characteristics, these are also very close to each other for certain ranges of the parameter values. In this paper, we intend to discriminate between the exponential and Lindley distribution functions considering the ratio of the maximized likelihood functions. The asymptotic distribution of the logarithm of the maximized likelihood ratio has been obtained to determine the minimum sample size required to discriminate between the two distributions for given probability of correct selection and a distance measure. Some numerical results are obtained to validate the asymptotic results. It is also observed that the asymptotic results work quite well even for small sample size. One data analysis is performed to demonstrate the results.

Keywords and Phrases: Asymptotic distribution; Likelihood ratio test; Probability of correct selection; Kolmogrov-Smirnov distance; Lindley distribution.

1 Introduction

The Lindley distribution with scale parameter θ, written as $Lin(\theta)$, having probability density function (pdf)

$$f_L(x;\theta) = \frac{\theta^2}{1+\theta}\,(1+x)\,e^{-\theta x};\ x,\theta > 0, \tag{1.1}$$

was introduced by Lindley [16]. The pdf is decreasing for $\theta \geq 1$ and is unimodal for $\theta < 1$. It is also known that the hazard function and the mean residual life (MRL) function of the distribution are increasing for all θ. Several aspects of the distribution are studied in detail by Ghitany *et al.* [9]. Lindley distribution being less popular among the univariate continuous

*Corresponding author e-mail: shovanc@iimk.ac.in; meetshovan@gmail.com

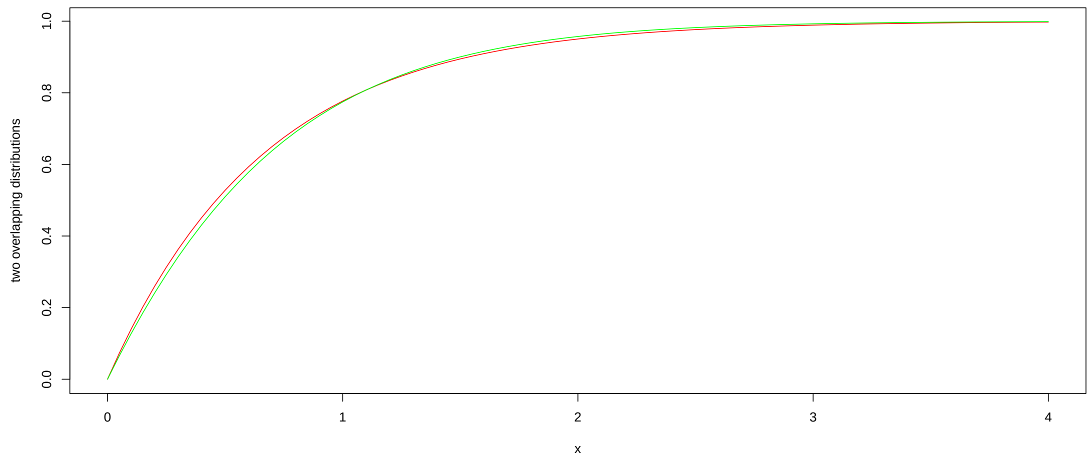

Figure 1: CDFs of $exp(1.5)$ and $Lin(2)$ distributions.

distributions has been overlooked in statistics literature. It is found that many of the mathematical properties of the Lindley distribution are more flexible than those of the exponential distribution.

Exponential distribution has been the most popular among the univariate continuous distributions with several significant statistical properties−most importantly, its characterization through lack of memory property. The exponential distribution with scale parameter λ, written as $exp(\lambda)$, has pdf given by

$$f_E(x;\lambda) = \lambda e^{-\lambda x};\ x, \lambda > 0.$$

While the pdf of the exponential is decreasing for all λ, the hazard and MRL functions are constant. This exhibits one of the distinguishable characteristics of the distributions. On the contrary, both the one parameter distributions can be quite effective in analyzing positively skewed data. Moreover, it is possible that pdfs or the cumulative distribution functions (cdfs) of both the distributions are very close to each other for certain ranges of the parameter values. Figure 1.1 shows the closeness of the cdfs of $exp(1.5)$ and $Lin(2)$ distributions claiming that one cdf can be used to fit the data while the data may come from the other pdf. This confirms that even if the exponential and the Lindley can be very close in the sense of a certain distributional characteristic, they may be quite different with respect to other characteristics. Although the two models may provide similar fit for small or moderate sample sizes, it is still important to choose the best fit for a given data set and to choose the correct model.

The problem of selecting the correct distribution is not new in the statistics literature. The problem for discriminating between two non-nested models was first considered by Cox [5, 6] and was later contributed by Bain and Engelhardt [2], Chen [4] and Fearn and Nebenzahl [8]. Due to the increasing applications of the lifetime distributions, special attention has been paid in selecting between the Weibull and log-normal distributions (Kundu and Manglick [12], Kim and Yum [11]), the gamma and log-normal distributions (Kundu and Manglick [13]), the generalized Rayleigh and log-normal distribution (Kundu and Raqab [14]), the generalized Rayleigh and Webull distribution (Raqab [18], Ahmad et al. [1]), the log-normal and generalized exponential distributions (Kundu et al. [15]), the Weibull and generalized exponential distributions (Gupta and Kundu [10]), the log-normal, weibull, and generalized exponential distributions (Dey and Kundu [7]), the exponential-Poisson and gamma distributions (Barreto-Souza, and Silva [3]), and the Poisson and geometric distributions (Pradhan and Kundu [17]).

In this paper, we consider the selection procedure of exponential and Lindley distributions. First, we use the logarithm of the ratio of maximized likelihoods (RML) to select the correct distribution. Then, we use the asymptotic behaviours of the logarithm of the RML to compute the probability of correct selection (PCS) to select the correct model. A comprehensive simulation is conducted to study the behaviour of the asymptotic results for different sample sizes. The rest of the paper is organized as follows. The asymptotic distributions of the logarithm of the RML statistics of the two distribution functions are obtained in Section 2. In Section 3, we determine the sample size required at a specified PCS which is used to discriminate between the two distributions. Numerical computations for PCS values based on the asymptotic results are presented in Section 4. One data analysis is presented for illustrative purpose in section 5.

2 Test statistic and asymptotic properties

In this section we describe the selection procedure on the basis of a random sample $X = \{x_1, x_2, ...x_n\}$. It is assumed that the data is generated from one of the $exp(\lambda)$ and $Lin(\theta)$ distributions and the corresponding likelihood functions are respectively

$$L_E\left(x;\lambda\right) = \prod_{i=1}^{n} f_E(x;\lambda),\ \ and\ L_L\left(x;\theta\right) = \prod_{i=1}^{n} f_L(x;\theta).$$

The RML is defined as $L = \frac{L_E(x;\lambda)}{L_L(x;\theta)}$; and $\hat{\lambda}$ and $\hat{\theta}$ are the maximum likelihood estimators of λ and θ respectively. Hence the logarithm of the RML, written as, $T = \log L = l_E(\hat{\lambda}) - l_L(\hat{\theta})$ is obtained as

$$T = n\left[\log\left(\frac{\hat{\lambda}\left(\hat{\theta}+1\right)}{\hat{\theta}^2}\right)\right] + (\hat{\theta}-\hat{\lambda})\sum_{i=1}^{n} x_i - \sum_{i=1}^{n}\log(1+x_i). \tag{2.1}$$

In case of exponential distribution, $\hat{\lambda}$ can be easily obtained as

$$\frac{n}{\sum_{i=1}^{n} x_i}.$$

Similarly, $\hat{\theta}$, the estimator of Lindley distribution can be obtained as

$$\hat{\theta} = \frac{-(\bar{x}-1) + \sqrt{(\bar{x}-1)^2 + 8\bar{x}}}{2\bar{x}}.$$

The natural model selection criterion will be to choose exponential distribution, if $T > 0$, otherwise, choose Lindley distribution.
Next, we derive the asymptotic distribution of T for two distinct cases, namely when the data are coming from $exp(\lambda)$ and $Lin(\theta)$ respectively. For the Borel measurable function $h(X)$, $E_E(h(X))$ and $V_E(h(X))$ will denote the mean and variance of $h(X)$ under the assumptions that X follows exponential distribution. Similarly, we define $E_L(h(X))$ and $V_L(h(X))$ as mean and variance of $h(X)$ under the assumption that X follows Lindley distribution. Moreover, if $g(X)$ and $h(X)$ are two Borel measurable functions, we define $Cov_E(g(U), h(U)) = E_E(g(U)h(U)) - E_E(g(U))E_E(h(U))$, and $Cov_L(g(U), h(U)) = E_L(g(U)h(U)) - E_L(g(U))E_L(h(U))$. Almost sure convergence will be denoted by $a.s.$ throughout the paper. We define the following notations for few integrals in the next subsections.

$$E_1(-\lambda) = \int_1^\infty \frac{e^{-\lambda t}}{t} dt,$$

$$\Lambda(i, j, k, l) = \int_0^\infty (\log(1 + x))^i (1 + x)^j e^{-(k\lambda + l\theta)x} dx.$$

2.1 Exponential distribution as the null hypothesis

We begin this section with the following Lemma. The proof of the Lemma follows using similar arguments of White [19] and hence is omitted.

Lemma 2.1 *Suppose the data are from $exp(\lambda)$ distribution. Then, as $n \to \infty$ we have that*

(i) $\hat{\lambda} \to \lambda$ *a.s.*

(ii) $\hat{\theta} \to \widetilde{\theta}$ *a.s., where*

$$E_E\left[\log f_L(x; \widetilde{\theta})\right] = \max_\theta E_E\left[\log f_L(x; \theta)\right],$$

where $\widetilde{\theta}$ is the quasi-likelihood estimators of θ

(iii) If $T^* = l_E(\hat{\lambda}) - l_L(\widetilde{\theta})$, then $n^{-1/2}(T - E_E T)$ is asymptotically equivalent to $n^{-1/2}(T^* - E_E T^*)$.

The following theorem follows from central limit theorem and Lemma 2.1(iii) and hence is omitted.

Theorem 2.1 *If the data are from $exp(\lambda)$ distribution, then T is approximately normally distributed with mean $E_E(T)$ and variance $V_E(T)$.*

Now, we discuss how to obtain $\widetilde{\theta}, E_E(T)$ and $V_E(T)$. Let us define

$$\begin{aligned}\Psi_E(\theta) &= E_E\left[\log f_L(x; \theta)\right] \\ &= 2\log\theta - \log(1 + \theta) - e^\lambda E_1(-\lambda) - \frac{\theta}{\lambda}.\end{aligned}$$

Differentiating $\Psi_E(\theta)$ with respect to θ (> 0), we get $\widetilde{\theta}$ after solving the following quadratic equation

$$\theta^2 + (1-\lambda)\theta - 2\lambda = 0.$$

We observe that $\lim_{n\to\infty} \frac{E_E(T)}{n}$ and $\lim_{n\to\infty} \frac{V_E(T)}{n}$ exist. Next we obtain the asymptotic mean and variance of T under $exp(\lambda)$ distribution which are denoted by $AM_E \approx \frac{E_E(T)}{n}$ and $AV_E \approx \frac{V_E(T)}{n}$ respectively and are derived as follows.

$$\begin{aligned} AM_E &= E_E\left[\log f_E(x;\lambda) - \log f_L(x;\widetilde{\theta})\right] \\ &= \log\lambda - 2\log\widetilde{\theta} + \log(1+\widetilde{\theta}) - \frac{\widetilde{\theta}}{\lambda} + e^{\lambda}E_i(-\lambda) - 1, \end{aligned}$$

and

$$\begin{aligned} AV_E &= V_E\left[\log f_E(x;\lambda) - \log f_L(x;\widetilde{\theta})\right] \\ &= \left(\widetilde{\theta}-\lambda\right)^2 V_E(X) + V_E\left(\log(1+X)\right) - 2(\widetilde{\theta}-\lambda)Cov_E\left(X, \log(1+X)\right) \\ &= \left(\frac{\widetilde{\theta}}{\lambda} - 1\right)^2 + \lambda\Lambda(2,0,1,0) + \left(e^{\lambda}E_1(-\lambda)\right)^2 - 2(\widetilde{\theta}-\lambda)\left(\lambda\Lambda(1,1,1,0) + (1+\frac{1}{\lambda})e^{\lambda}E_1(-\lambda)\right). \end{aligned}$$

2.2 Lindley distribution as the null hypothesis

Along the same line as Lemma 2.1 and Theorem 2.1, we state the following results.

Lemma 2.2 *Suppose the data are from $Lin(\theta)$ distribution. Then, as $n \to \infty$ we have that*

(i) $\hat{\theta} \to \theta$ *a.s.*

(ii) $\hat{\lambda} \to \widetilde{\lambda}$ *a.s. where*

$$E_L\left[\log f_E(x;\widetilde{\lambda})\right] = \max_{\lambda} E_L\left[\log f_E(x;\lambda)\right],$$

where $\widetilde{\lambda}$ is the quasi-likelihood estimators of λ

(iii) If $T_* = l_E(\widetilde{\lambda}) - l_L(\hat{\theta})$, then $n^{-1/2}(T - E_L T)$ is asymptotically equivalent to $n^{-1/2}(T_* - E_L T_*)$.

As mentioned earlier the following theorem follows from central limit theorem and Lemma 2.2(iii) and hence is omitted.

Theorem 2.2 *If the data are from $Lin(\theta)$ distribution, then T is approximately normally distributed with mean $E_L(T)$ and variance $V_L(T)$.*

To obtain $\widetilde{\lambda}, E_L(T)$ and $V_L(T)$. Let us define

$$\begin{aligned} \Psi_L(\lambda) &= E_L\left[\log f_E(x;\lambda)\right] \\ &= \log\lambda - \frac{\lambda(\theta+2)}{\theta(\theta+1)}. \end{aligned}$$

λ	$AM_E(\lambda)$	$AM_V(\lambda)$	$\widetilde{\theta}$
0.1	0.0773	0.1952	0.184
0.5	0.0175	0.0401	0.781
0.9	0.0073	0.0162	1.292
1.3	0.0038	0.0083	1.769
1.5	0.0028	0.0062	2.000
2.0	0.0016	0.0033	2.561
2.5	0.0009	0.0020	3.108

Table 1: Different values of $AM_E(\lambda), AV_E(\lambda), \widetilde{\theta}$ for different values of λ

Differentiating $\Psi_L(\lambda)$ with respect to λ, we get

$$\widetilde{\lambda} = \frac{\theta(\theta+1)}{\theta+2}.$$

As before we obtain the expressions of asymptotic mean and variance of T under $Lin(\theta)$ distribution as $AM_L \approx \frac{E_L(T)}{n}$ and are derived as

$$\begin{aligned} AM_L &= E_E\left[\log f_E(x;\widetilde{\lambda}) - \log f_L(x;\theta)\right] \\ &= \log\widetilde{\lambda} - (\widetilde{\lambda}-\theta)\frac{\theta+2}{\theta(\theta+1)} + \log\left(\frac{1}{\theta}+\frac{1}{\theta^2}\right) - \frac{\theta^2}{\theta+1}\Lambda(1,1,0,1), \end{aligned}$$

and

$$\begin{aligned} AV_L &= V_L\left[\log f_E(x;\widetilde{\lambda}) - \log f_L(x;\theta)\right] \\ &= \left(\theta-\widetilde{\lambda}\right)^2 V_L(X) + V_L\left(\log(1+X)\right) - 2(\theta-\widetilde{\lambda})Cov_L\left(X,\log(1+X)\right) \\ &= \left(\theta-\widetilde{\lambda}\right)^2 \frac{\theta^2+4\theta+2}{\theta^2(\theta+1)^2} + \frac{\theta^2}{\theta+1}\Lambda(2,1,0,1) - \left(\frac{\theta^2}{\theta+1}\Lambda(1,1,0,1)\right)^2 - \\ &\quad 2(\theta-\widetilde{\lambda})\left(\left(\frac{\theta^2}{\theta+1}\left(\Lambda(1,2,0,1)-\Lambda(1,1,0,1)\right)\right) - \frac{\theta(\theta+2)}{(\theta+1)^2}\Lambda(1,1,0,1)\right). \end{aligned}$$

3 Selection procedure

In this section, we will make a choice between exponential and Lindley distributions for which we determine minimum sample size for a given probability of correct selection (PCS) and tolerance limits which can be measured through the distance between two cumulative distribution functions (cdf). Practically, the tolerance limit measures the closeness between two cdfs. It is obvious that if the distance between two cdfs is very small, one needs a very large sample size to discriminate between them for a given PCS. On the other hand if the cdfs

θ	$AM_L(\theta)$	$AM_L(\theta)$	$\widetilde{\lambda}$
0.1	-0.0802	0.1203	0.052
0.5	-0.0275	0.0479	0.300
0.9	-0.0124	0.0223	0.589
1.3	-0.0066	0.0121	0.906
1.5	-0.0049	0.0092	1.071
2.0	-0.0026	0.0049	1.500
2.5	-0.0016	0.0029	1.944

Table 2: Different values of $AM_L(\theta), AV_L(\theta), \widetilde{\lambda}$ for different θ

are far apart, moderate to small sample size may be sufficient to discriminate between the two for a given PCS. Here we use Kolmogrov−Smirnov (K−S) distance to discriminate between the two cdfs with K−S distance being defined as $\sup_x |F(x) - G(x)|$, where F and G are the cdf of exponential and Lindley distributions respectively. One may use other distance measures with the same selection criterion. So, minimum sample size can be determined based on the given PCS (p, say) and the tolerance limit (D, say) as described in the next subsection.

3.1 Determination of sample size

In view of Theorem 2.1, T is asymptotically normally distributed with mean $E_E(T)$ and variance $V_E(T)$,. The PCS for selecting exponential distribution is given by

$$PCS(\lambda) = P(T > 0 \mid \lambda) \approx \Phi\left(\frac{E_E(T)}{\sqrt{V_E(T)}}\right) = \Phi\left(\frac{\sqrt{n}AM_E(T)}{\sqrt{AV_E(T)}}\right),$$

where Φ denotes the cdf of the standard normal random variable. Sample size can be determined by equating the PCS(λ) to the given protection level p as given by

$$\Phi\left(\frac{\sqrt{n}AM_E(T)}{\sqrt{AV_E(T)}}\right) = p$$

to get

$$n = \frac{z_p^2 AV_E(T)}{(AM_E(T))^2}.$$

For $p = 0.6, 0.7, 0.8$ sample size and K−S distance are reported in Table 3 for different values of λ. Proceeding in the similar manner, using Theorem 2.2 sample size can be determined as

$$n = \frac{z_p^2 AV_L(T)}{(AM_L(T))^2}.$$

For $p = 0.6, 0.7, 0.8$ sample size and K−S distance are reported in Table 4 for different choices of θ. Here z_p is the $100p$ percentile point of a standard normal distribution.

$\lambda \rightarrow$	0.1	0.5	0.9	1.3	1.5	2.0	2.5
n ($p = 0.6$)	2	8	20	37	49	88	144
n ($p = 0.7$)	9	36	84	159	208	375	617
n ($p = 0.8$)	23	93	216	408	536	967	1590
K-S	0.106	0.054	0.034	0.027	0.021	0.018	0.012

Table 3: Values of n and K$-$S distances between $exp(\lambda)$ and $Lin(\widetilde{\theta})$ distributions.

$\sigma \rightarrow$	0.1	0.5	0.9	1.3	1.5	2.0	2.5
n ($p = 0.6$)	2	4	9	18	24	45	77
n ($p = 0.7$)	23	17	40	77	103	194	332
n ($p = 0.8$)	53	45	103	199	265	499	855
K-S	0.120	0.070	0.049	0.036	0.031	0.022	0.015

Table 4: Values of n and K$-$S distances between $Lin(\theta)$ and $exp(\widetilde{\lambda})$.

We shall now discuss how to use the PCS and the tolerance level to discriminate between exponential and Lindley models. Suppose the data are from exponential cdf. Further, suppose that the tolerance level is based on the K$-$S distance and is fixed at 0.054, and the protection level $p = 0.8$. Here tolerance level $D = 0.054$ means that the practitioner wants to discriminate between exponential and Lindley cdfs only when their K$-$S distance is more than 0.054. Table 3 shows that one needs to take a sample of size 93 for $p = 0.8$ to discriminate the exponential and Lindley distributions. When the data are from Lindley distribution, for $D = 0.054$, and $p = 0.8$, Table 4 gives the minimum value of $n = 45$. Therefore, for the given tolerance level of 0.054, one needs a sample of size max(93,45)=93 to meet the protection level $p = 0.8$ simultaneously for both the cases.

4 Numerical results

In this section we will show that the asymptotic results derived in Section 3 work well for finite sample sizes. We compute the PCS based on asymptotic results derived in Section 3. Sample of size n =20, 40, 60, 80, 100, and 200 are taken for the findings. First we consider the case when the null distribution is exponential and the alternative is Lindley. The results obtained by using the asymptotic theory are shown in Table 5 for various choices of the scale parameter of exponential distribution viz. $\lambda = 0.1, 0.5, 0.9, 1.3, 1.5, 2.0, 2.5$.
Similarly, we obtain the results for the same choice of n and the scale parameter of Lindley distribution θ when the null distribution is Lindley and the alternative is exponential. The results are reported in Table 6. It is clear from Tables 5 and 6 that as the sample size increases

$\alpha \downarrow \; n \rightarrow$	20	40	60	80	100	200
0.1	0.783	0.864	0.912	0.941	0.959	0.993
0.5	0.652	0.709	0.750	0.782	0.808	0.891
0.9	0.601	0.641	0.671	0.696	0.716	0.791
1.3	0.574	0.604	0.626	0.645	0.661	0.722
1.5	0.567	0.590	0.612	0.634	0.648	0.701
2.0	0.553	0.575	0.584	0.608	0.617	0.655
2.5	0.537	0.553	0.564	0.575	0.583	0.617

Table 5: PCS based on asymptotic results when the data are from $exp(\lambda)$ distribution.

$\theta \downarrow \; n \rightarrow$	20	40	60	80	100	200
0.1	0.852	0.930	0.966	0.983	0.988	0.990
0.5	0.714	0.786	0.832	0.861	0.891	0.965
0.9	0.644	0.704	0.748	0.773	0.809	0.882
1.3	0.601	0.656	0.683	0.702	0.727	0.804
1.5	0.598	0.632	0.655	0.683	0.702	0.776
2.0	0.577	0.591	0.613	0.634	0.657	0.708
2.5	0.555	0.579	0.594	0.606	0.615	0.669

Table 6: PCS based on based on asymptotic results when the data are from $Lin(\theta)$ distribution.

the PCS also increases as expected. It is also observed that the PCS increases as the value of λ and θ decreases. Moreover, asymptotic results work quite well when the sample size is as small as 20 in both the cases for all possible parameter ranges.

5 Data Analysis

In this section, we use a real data set to select between exponential and Lindley distributions. The data set as furnished in Ghitany *et al.* [9] represents the waiting times (in minutes) before service of 100 bank customers. The maximum likelihood estimates of λ and θ are computed as $\hat{\lambda} = 0.101$ and $\hat{\theta} = 0.187$.

When exponential and Lindley distributions are used to fit the data, maximized log-likelihhod functions are given as $l_E(0.101) = -329$ and $l_L(0.187) = -319$ respectively resulting in $T = l_E(0.101) - l_L(0.187) = -329 + 319 = -19 < 0$ which indicates to choose the Lindley model. Now we compute the PCS based on asymptotic results. Assuming that the waiting time data are from exponential cdf, the asymptotic mean and variance are obtained as $AM_E(0.101) = 0.0768$ and $AV_E(0.101) = 0.1940$ along with the $PCS = 0.959$ yielding an estimated risk less than around four percent to choose the wrong model. Similarly, assuming that the data are from Lindley cdf, we compute $AM_L(0.187) = -0.0611$ and $AV_L(0.187) = 0.0976$, with the $PCS = 0.975$ to yield an estimated risk less than approximately two percent to choose the wrong model. Therefore, the PCS is at least min(0.959,0.976)=0.959 in this case. The PCS attains the maxima when the data is coming from Lindley distribution and hence we should choose Lindley distribution to fit the waiting time data.

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Research Office

Indian Institute of Management Kozhikode

IIMK Campus P. O.,

Kozhikode, Kerala, India,

PIN - 673 570

Phone: +91-495-2809237/ 238

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